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THE PREDICTION OF AIRFLOW-GENERATED NOISE IN DUCTS FROM CONSIDERATIONS OF SIMILARITY

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1. INTRODUCTION

In recent years, there have been a number of attempts to devise generalized prediction techniques for airflow-generated noise [1]. A technique based upon the static pressure loss due to in-duct elements which built upon the earlier work of Nelson and Morfey [2] has recently been proposed by the authors [3]. In this paper, we describe how the Nelson and Morfey work can also be employed as the starting point of an alternative technique based upon aerodynamic and acoustical similarity.

Many fluid-flow situations are of such complexity that they cannot be modelled using analytical techniques. An alternative approach is to use data measured on one configuration to predict the behaviour of another configuration which is sufficiently similar for simple scaling laws to be applied. For example, this is the basis of the wind-tunnel testing of scale models to determine the drag characteristics of dynamic objects. In employing similarity in this way, it is necessary to identify relevant dimensionless groupings of parameters. The Strouhal number is such a dimensionless grouping and its most well-known application is to relate the frequency of vortex shedding from a cylindrical spoiler in an air stream to the velocity of the airflow and the diameter of the spoiler. The Strouhal number is also frequently employed to normalize spectral data relating to the noise generated by in-duct flow spoilers (see, for example, reference [4]).

In this paper, we show how similarity can be used to predict the airflow noise generated by common types of duct components.

2. THE GOVERNING EQUATIONS

Nelson and Morfey [2] devised an expression for the sound power radiated from an in-duct spoiler subjected to the action of fluctuating drag forces arising from the flow turbulence in the vicinity of the spoiler. The noise generated by an in-duct flow spoiler was treated by replacing the spoiler and the turbulence by an equivalent acoustic distribution of dipole sources radiating into a duct filled with fluid at rest. They obtained two approximate expressions for the sound power of regenerated noise from the solution for the dipole source of the inhomogeneous wave equation governing the propagation of sound in an infinite hard-walled duct. One expression applies to frequencies below the cut-on frequency f_0 of the

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first transverse duct mode (plane wave propagation only), the other one applies to frequencies above the cut-on frequency f_0 (multi-mode propagation).

The sound power $W_{\Delta f}$ radiated in a given bandwidth Δ_f is given as follows:
for $f_c < f_0$,

$$W_{\Delta f} \approx (1/4A\rho_0 c_0)(\bar{F}_3^2)_{\Delta f} \tag{1}$$

and for $f_c > f_0$,

$$W_{\Delta f} \approx (\omega_c^2/24\pi\rho_0 c_0^3)(\bar{F}_3^2)_{\Delta f}[1 + (3\pi c_0/4\omega_c)(a+b)/A], \tag{2}$$

where A is the cross-sectional area of the duct, ρ_0 is the density of air, c_0 is the speed of sound in air, $(\bar{F}_3^2)_{\Delta f}$ is the mean square of the fluctuating force in a given band, f_c is the centre frequency of the band of frequencies under consideration, f_0 is the cut-on frequency of the duct, ω_c is the angular centre frequency, a is the cross-sectional duct width, and b is the duct height.

However, as it was impossible to quantify the fluctuating drag forces, Nelson and Morfey developed their predictive technique by assuming that the root-mean-square fluctuating force acting on the spoiler is directly proportional to the steady-state drag force to yield the following equations:
for $f_c < f_0$,

$$W_{\Delta f} = (\rho_0/16c_0)AK^2(\mathrm{St})[\sigma^2(1-\sigma)]^2 C_D^2 U_C^4 \tag{3}$$

and for $f_c > f_0$,

$$W_{\Delta f} = (\rho_0\pi/24c_0^3)[1 + (3\pi c_0/4\omega_c)(a+b)/A](A/d)^2(\mathrm{St})^2 K^2(\mathrm{St})[\sigma^2(1-\sigma)]^2 C_D^2 U_C^6, \tag{4}$$

where C_D is the drag coefficient, σ is the open area ratio, U is the flow velocity, $U_C = U/\sigma$ is the maximum effective velocity, d is the characteristic dimension, $\mathrm{St} = fd/U_c$ is the Strouhal number, and $K(\mathrm{St})$ is a Strouhal-number-dependent factor. The drag coefficient C_D is a dimensionless quantity which will be constant if measured on geometrically similar configurations over the range of Reynold's numbers encountered in ventilation ducts. Thus, it represents the starting point for the procedure to be outlined in this paper. Although Nelson and Morfey worked with simple strip spoilers in square ducts, Oldham and Ukpoho [4] have shown that by treating more complex arrangements in terms of simple strips, their basic equations can be successfully applied to these configurations in both circular and rectangular ductwork. Waddington and Oldham [3] have further demonstrated that the Nelson and Morfey approach can be applied to actual duct components such as bends.

3. THE PREDICTIVE TECHNIQUE

3.1. BELOW THE CUT-ON FREQUENCY

On examination of equation (3), it can be noted that both the open area ratio σ and the drag coefficient C_D are dimensionless and functions of the geometry of the noise-generating element. In addition, for the conditions encountered in a normal ventilation system, the density of air ρ_0 and the velocity of sound in air c_0 will be constants. The remaining terms are the Strouhal-number-dependent factor of proportionality $K(\mathrm{St})$ and the constriction velocity U_C, which is simply the mean duct velocity U divided by the open area ratio σ.

The Strouhal number is given by $\mathrm{St} = fd/U_c$ where d is a characteristic dimension of the noise-generating element. Apart from the case of a simple duct element such as a strip spoiler, it is not readily apparent how this dimension should be defined. However, as it will be a function of the duct geometry in the vicinity of the noise-generating element, the ratio of characteristic dimensions for two geometrically similar elements will be equal to the ratio of the linear dimensions of any other duct dimension.

For the case of a given duct element, the ratio of the sound power level generated at fixed Strouhal number for different air velocities is simply proportional to the fourth power of the air velocities. As d (being a function of geometry) does not change then, in order to maintain a constant value of Strouhal number, if the air velocity changes there must be a corresponding change in frequency in proportion to the change in velocity.

Thus, given the in-duct sound power level spectrum resulting from one velocity condition for a particular duct component, equation (3) can be used to obtain spectra for noise generated by the same component for different flow velocities.

As stated above, the drag coefficient and open area ratio will be constant for different sizes of geometrically similar elements. It can also be seen from equation (3) that for a geometrically similar duct element at a given velocity and for a constant value of Strouhal number, the acoustic power generated by the two elements is simply proportional to their cross-sectional areas. In this case, the change in cross-sectional area will result in a change in the characteristic dimension, which is in proportion to the square root of the change in cross-sectional area. Thus, in order to maintain the value of Strouhal number there must again be a change in frequency in inverse proportion to the square root of the change in cross-sectional area. A change in cross-sectional area will also result in a change in the cut-on frequency in inverse proportion to the square root of the change in cross-sectional area.

It should, therefore, be possible, using the procedures described above, to use the spectral data corresponding to a particular size of component and one airflow velocity to predict the sound power level spectrum for a different-sized component for a range of air velocities.

The resulting predictive equations are as follows:
for $f_m < f_{0p}$

$$W(f_p) = W(f_m) + 40\log\left(\frac{U_p}{U_m}\right) + 10\log\left(\frac{A_m}{A_p}\right), \tag{5}$$

where $W(f_p)$ is the sound power level at frequency f_p, and

$$f_p = f_m\left(\frac{U_p}{U_m}\right)\left(\frac{A_m}{A_p}\right)^{1/2}, \tag{6}$$

where $W(f_m)$ is the measured sound power level at frequency f_m, U_m is the measured air velocity, U_p is the predicted air velocity, A_m is the original cross-sectional area, A_p is the cross-sectional area of the new duct, and

$$f_{0p} = f_0\left(\frac{A_m}{A_p}\right)^{1/2}, \tag{7}$$

where f_{0p} is the cut-on frequency of the new duct and f_0 is the original cut-on frequency.

3.2. ABOVE THE CUT-ON FREQUENCY

Equation (4) is slightly more complex than equation (3) but it can be applied in a similar manner. There is, however, an apparent complication arising from the following term within the equation: $[1 + (3\pi c_0/4\omega_c)(a + b)/A]$. This term effectively bridges the transition between the v^4 and the v^6 velocity-dependant regions. It can be shown [1] that for a square-section duct, this expression reduces to $[1 + 3f_0/4f_c]$ which is again a dimensionless quantity. At the cut-on frequency, this term contributes less than 2·5 dB to the predicted sound power level whilst at higher frequencies its contribution becomes negligible. A similar expression can be derived for circular ductwork to give $[1 + 1{\cdot}3f_0/f_c]$.

From an examination of equation (4) it can be seen that for a given duct element, the ratio of the sound power level generated at fixed Strouhal number for different air velocities is proportional to the sixth power of the air velocities. As the characteristic dimension d does not change then, in order to maintain a constant value of Strouhal number, if the air velocity changes there must be a corresponding change in frequency as described in section 3.1.

For the case of a geometrically similar duct element it can be seen from equation (4) that at a given velocity and for a constant value of Strouhal number, the acoustic power generated by the two elements is again simply proportional to their cross-sectional areas. However, in order to predict the sound power level spectra, it is necessary to take into account the fact that the change in cross-sectional area will result in a change in the characteristic dimension. Thus, in order to maintain the value of Strouhal number there must again be a change in frequency as described in section 3.1.

The resulting predictive equation for square-section ductwork is given as follows:
for $f_m > f_{0p}$

$$W(f_p) = W(f_m) + 60\log\left(\frac{U_p}{U_m}\right) + 10\log\left(\frac{A_m}{A_p}\right) + 10\log\left[\frac{(1 + 3f_{p0}/4f_m)}{(1 + 3f_0/4f_m)}\right]. \tag{8}$$

4. SOUND POWER LEVEL DATA

The predictive techniques described above were applied to three different sizes of circular ductwork with long radius bends. The authors have been fortunate to acquire data obtained as part of a comprehensive series of measurements undertaken several years ago by Atkins Noise and Vibration of Epsom, U.K. The sound power level spectra were determined from sound pressure levels measured in a reverberation chamber into which the duct containing the test element were fed [5].

4.1. COMPONENT WITH DIFFERENT AIR VELOCITIES

An example of the application of the above procedure can be seen in Figures 1 and 2 which show in-duct sound power spectra corresponding to a range of air velocities in long radius bends in 350 and 200 mm diameter circular ductwork. The second spectrum has been employed to predict spectra for four other air velocities. These are shown with corresponding experimental data. It can be seen that the agreement between the predicted and experimental data is generally good.

As the predictive curves have been produced using experimental data, they have the same degree of uncertainty as the experimental curves. Although no information is available regarding the actual scatter associated with the measured data, it is generally accepted that

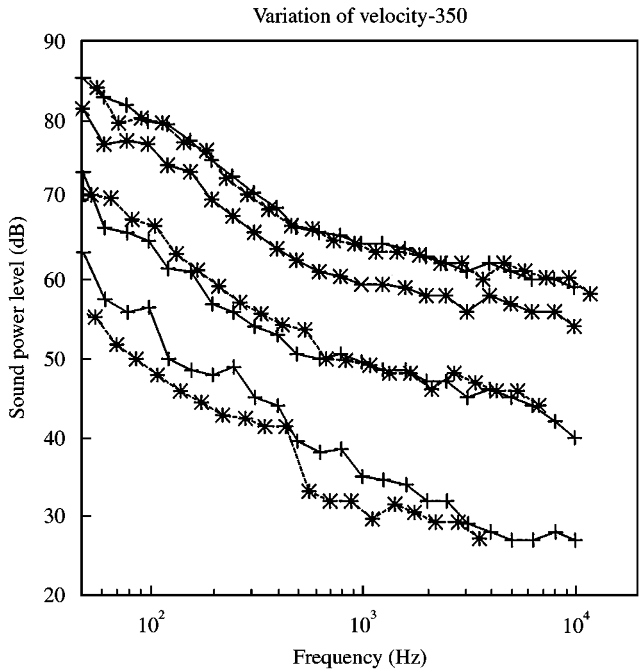

Figure 1. Comparison of measured (+) and predicted (*) flow noise spectra for long radius bend in 350 mm diameter circular duct for different air velocities.

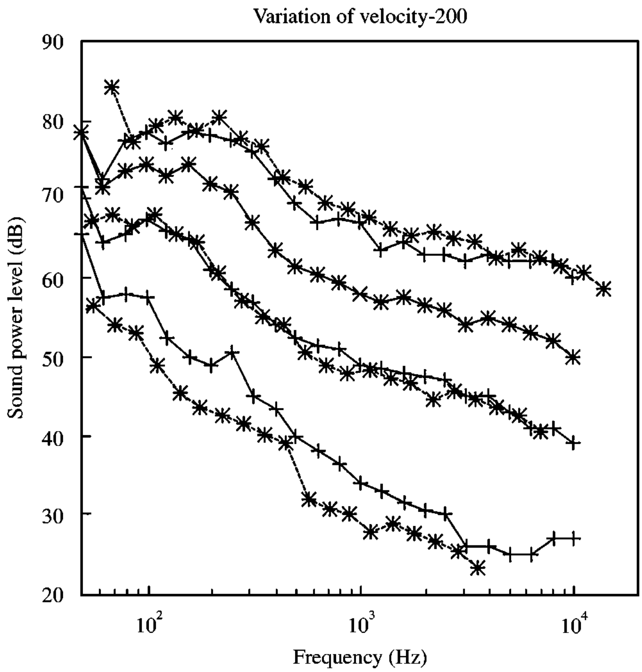

Figure 2. Comparison of measured (+) and predicted (*) flow noise spectra for long radius bend in 200 mm diameter circular duct for different air velocities.

measurements of sound power level made using the reverberation chamber method involve uncertainties of ± 2–3 dB at the lowest and highest frequencies and $\approx \pm 1$ dB at the midfrequencies [2, 4].

The agreement is least good for the spectra corresponding to the lowest air velocity. This may be due to the low levels of noise generated at this velocity which are comparable to the reported system background noise present during the measurement programme [5]. In addition, the experimental uncertainties associated with the measurement of low air speeds are greater than those associated with higher speeds and these will affect the noise predictions for low air velocities.

The data indicate the potential limitation of the technique due to the limited bandwidth of the measured spectrum. This results in there being no data from which: (i) to extrapolate at the low-frequency end of the spectrum when considering a velocity lower than that of the measured data, or (ii) to extrapolate at the high-frequency end of the spectrum for velocities higher than that of the measured spectrum. Thus, the degree of uncertainty will increase as one attempts to predict noise levels for velocities which differ by increasingly greater amounts from that used as the basis of the prediction.

4.2. GEOMETRICALLY SIMILAR COMPONENTS WITH DIFFERENT AIR VELOCITIES

Figure 3 shows the in-duct spectra corresponding to a long radius bend in 200 mm diameter circular ductwork for a range of different flow velocities. The spectra displayed are predicted by the technique described above using experimental data for the 350 mm diameter duct. Also shown are the data obtained from experimental measurements.

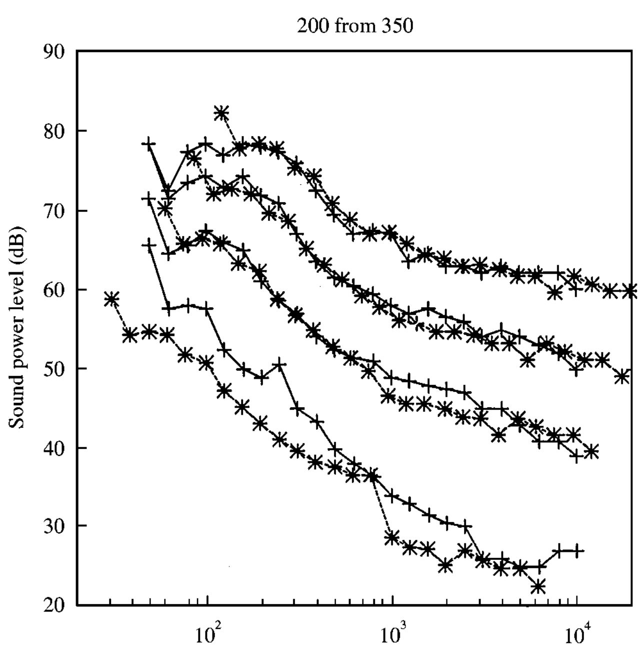

Figure 3. Comparison of measured (+) and predicted (*) flow noise spectra for long radius bend in 200 mm diameter circular duct from data measured on 350 mm diameter duct.

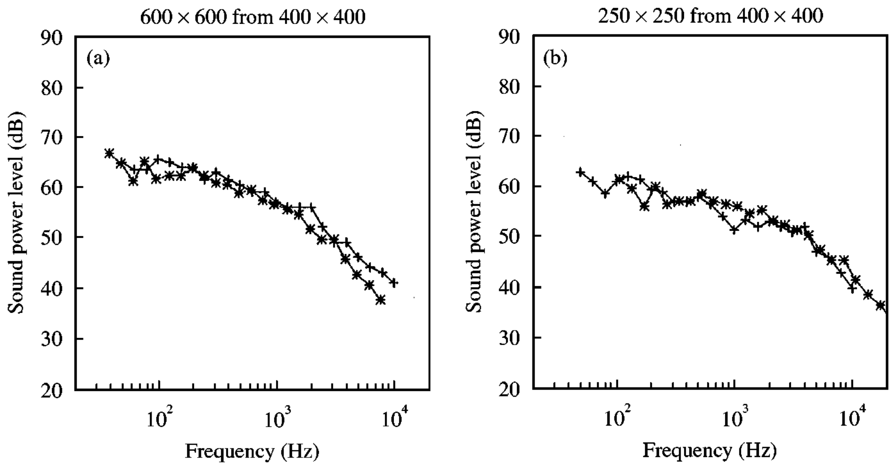

Figure 4. Comparison of measured (+) and predicted (*) flow noise spectra for mitred bend in 600 and 250 mm square ducts from data measured on 400 mm square duct.

The agreement between the predicted and measured data is generally good apart from that at the lowest air velocities. As discussed above, the low-velocity data are likely to be adversely affected by background noise and experimental uncertainties associated with the measurement of low air velocities. The data again indicate the potential limitation of the technique due to the restricted bandwidth of the measured spectrum. Figure 4 shows a comparison of predicted and measured data for mitred bends in 250 and 600 mm square-section ductwork. The predicted values have been calculated from measured data on 400 mm square-section ductwork. The agreement can again be seen to be good.

5. CONCLUSIONS

It has been shown that by consideration of similarity, the Nelson and Morfey equations can be employed as the basis of a prediction technique for the noise generation due to the interaction of airflow and duct elements. Although, in principle, data from a single measurement can be employed for a large range of air velocities and duct dimensions, the uncertainty associated with the predictions increases for situations which differ greatly from those used as the basis for the predictions.

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